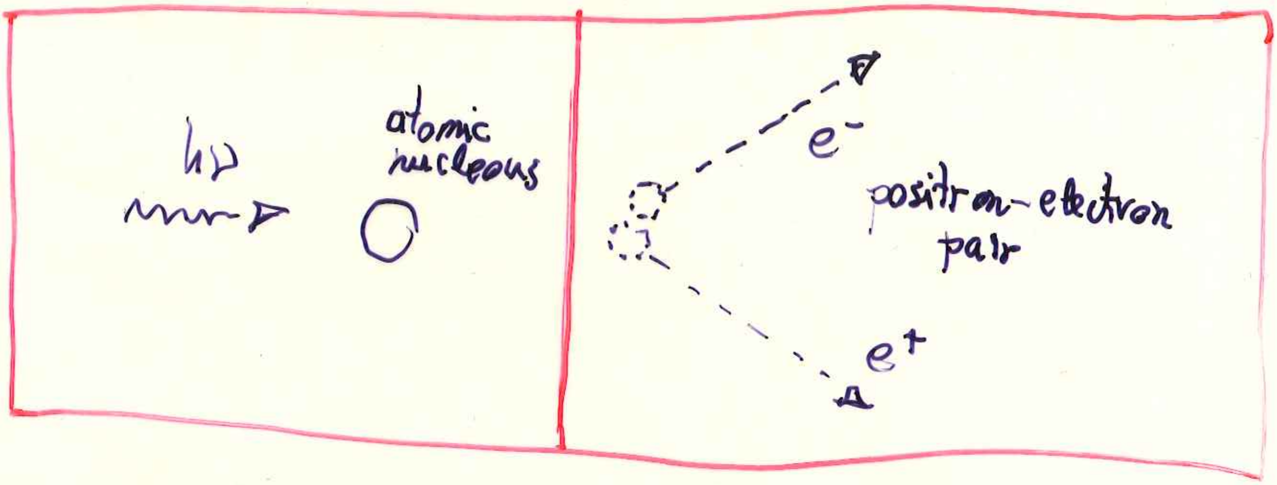
Pair production



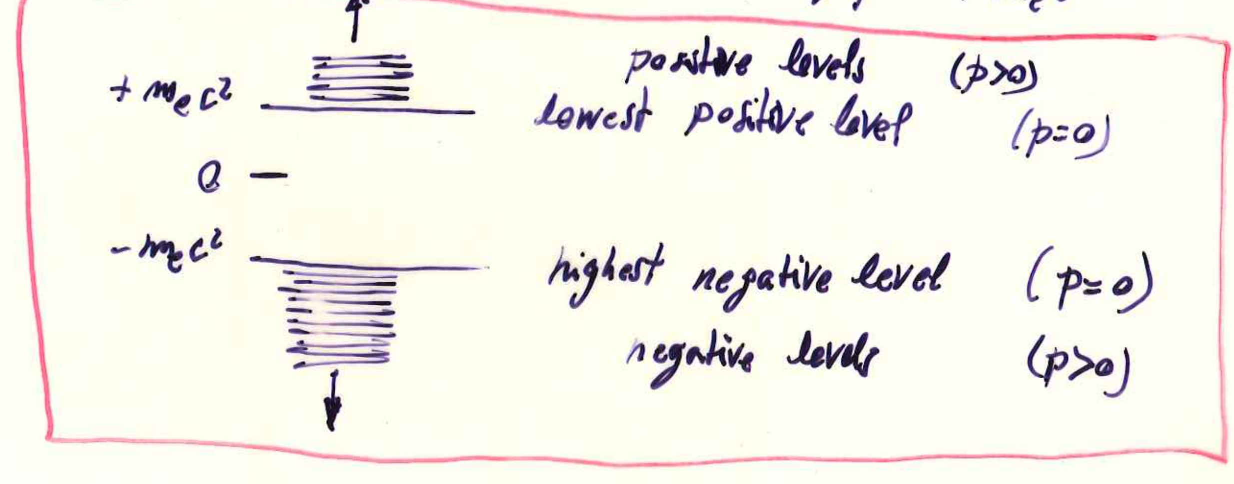
- photon energy $h\nu$ is completely absorbed
- the interaction requires a minimum energy $h\nu = 2\,m_e c^2$
  $\simeq 1.022$ MeV
- Energy conservation gives

$$h\nu = (T_- + m_e c^2) + (T_+ + m_e c^2)$$

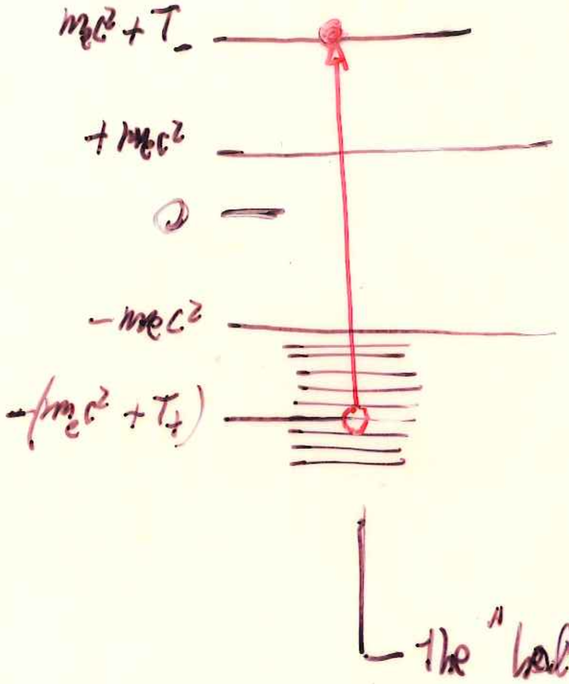
$$T_- + T_+ = h\nu - 2\,m_e c^2$$

- pair production confirmed Dirac theory of the electron

$$E^2 = p^2c^2 + m_e c^2 \Rightarrow E = \pm\sqrt{p^2c^2 + m_e c^2}$$



Dirac's Empty space ⇒ formed by electrons with negative energy (filled levels)



} fully occupied negative energy states for electrons

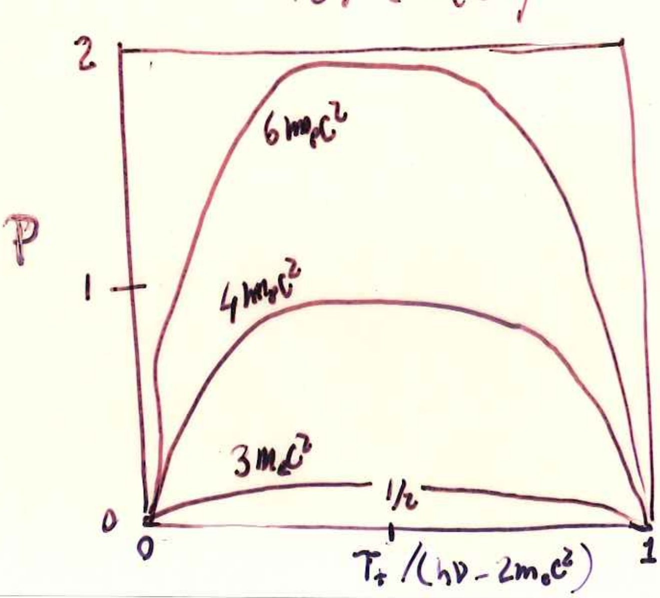
the "hole" in the negative energy levels represents the 'positron'.

Energy distribution of pairs produced electrons

$$d\,{}_a\kappa = \frac{\sigma_0 Z^2 P}{h\nu - 2m_ec^2}\, dT_+$$

where

$$\sigma_0 = \frac{1}{137}\left(\frac{e^2}{m_ec^2}\right)^2 = 5{,}8\ 10^{-28}\ \frac{cm^2}{nucleous}$$



Total pair production cross section

$$ {}_a k = \int d({}_a k) = \sigma_0 Z^2 \int_0^{h\nu - 2m_ec^2} \frac{P\, dT_+}{h\nu - 2m_ec^2} $$

⇓

$$ {}_a k = \sigma_0 Z^2 \int_0^1 P\, d\left[\frac{T_+}{h\nu - 2m_ec^2}\right] $$

$$ = \sigma_0 Z^2 \bar{P} \quad \left[\frac{cm^2}{nucleons}\right] $$

Total Attenuation Coefficient

probability of traversing a thickness x without a:

Compton collision → $e^{-\sigma_c x}$

Rayleigh collision → $e^{-\sigma_R x}$

Photoelectric collision → $e^{-\tau x}$

Pair-production collision → $e^{-\kappa x}$

A collimated x-ray beam of initial intensity I_0 after traversing a thickness x of absorber will have a residual intensity I

$$I = I_0 e^{-\sigma_c x} e^{-\sigma_R x} e^{-\tau x} e^{-\kappa x}$$

$$= I_0 e^{-(\sigma_c + \sigma_R + \tau + \kappa) x}$$

$$= I_0 e^{-\mu x}$$

where μ is the total attenuation coefficient

$$\boxed{\mu = \sigma_c + \sigma_R + \tau + \kappa}$$

Libero cammino medio

Def: distanza media che un fotone può percorrere (in un mezzo assorbente) senza essere assorbito

[Si esprime in termini del coefficiente di assorbimento]

Spessore ottico

$$dL_\nu = \mu_\nu \, ds$$

L spessore ottico

$$\Rightarrow L_\nu = \int_{s_0}^{s} \mu_\nu(s')\, ds' \quad \Rightarrow \begin{cases} L_\nu > 1 & \text{mezzo opaco} \\ L_\nu < 1 & \text{mezzo trasparente} \end{cases}$$

s_0 un punto arbitrario definisce lo zero della scala

e^{-L_ν} → probabilità di percorrere uno spessore ottico L_ν

Calcoliamo lo spessore medio percorso

$$\langle L_\nu \rangle = \int_0^\infty L_\nu e^{-L_\nu} dL_\nu$$

$$= 1$$ (Si può dimostrare calcolando l'integrale)

$$\int_0^\infty L_\nu e^{-L_\nu} dL_\nu = (-L_\nu - 1) e^{-L_\nu} \Big|_0^\infty$$

$$= 1$$

Libero cammino medio ℓ_ν è dato da:

$$\langle L_\nu \rangle = \ell_\nu \mu_\nu = 1$$

$$\Longrightarrow \boxed{\ell_\nu = \frac{1}{\mu_\nu}}$$

Energy absorption

primary ionization is produced by a photon when it removes an electron from an atom by
(1) photoelectric effect
(2) Compton collision

Energy of the secondary electron

Photoelectric effect $E = h\nu - E_{characteristic}$

Compton scattering $E = h\nu - h\nu'$

⟹ Most of the energy of the primary photon is transferred to the secondary electron

Energy is released (by secondary electrons) through two mechanisms:

(a) bremsstrahlung — small fraction ~ few %

(b) ionization ← Secondary ionization
(~ 32 eV per ion-pair produced)

Example: 1 MeV electron → ~ 30.000 ionpairs

primary ionization 1 (negligible)

Secondary ionization 30 000