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Anomalous Rayleigh scatter in dilute media

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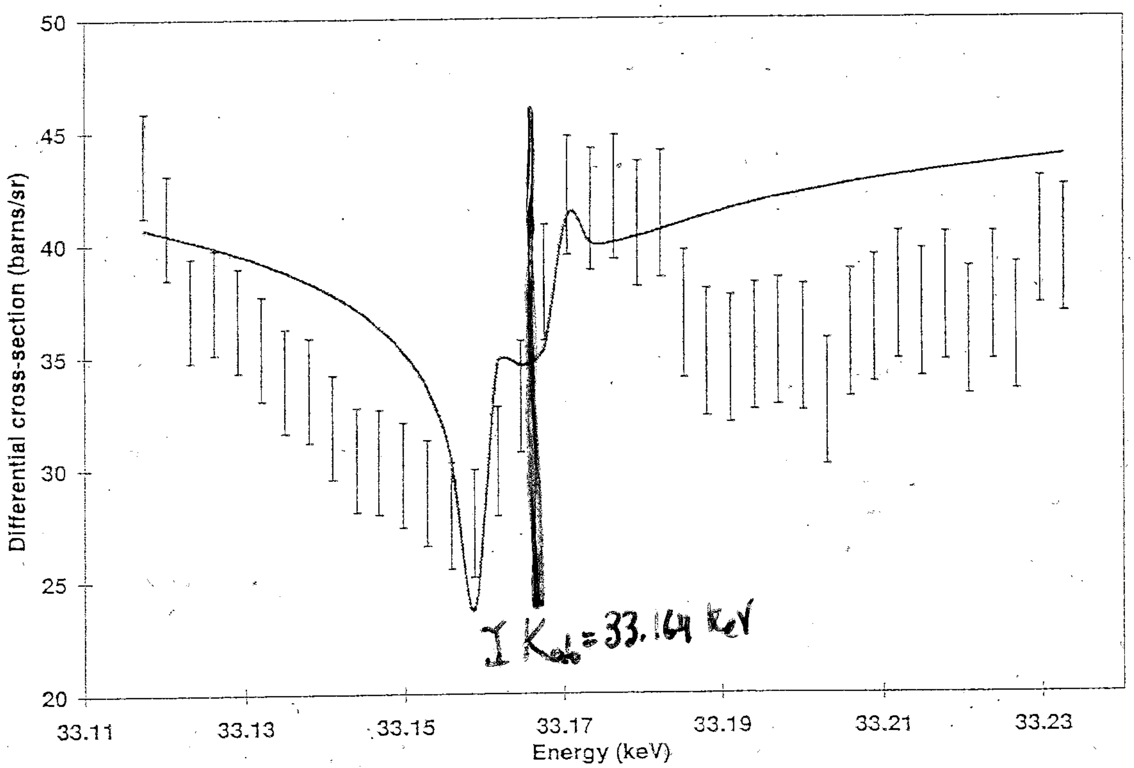
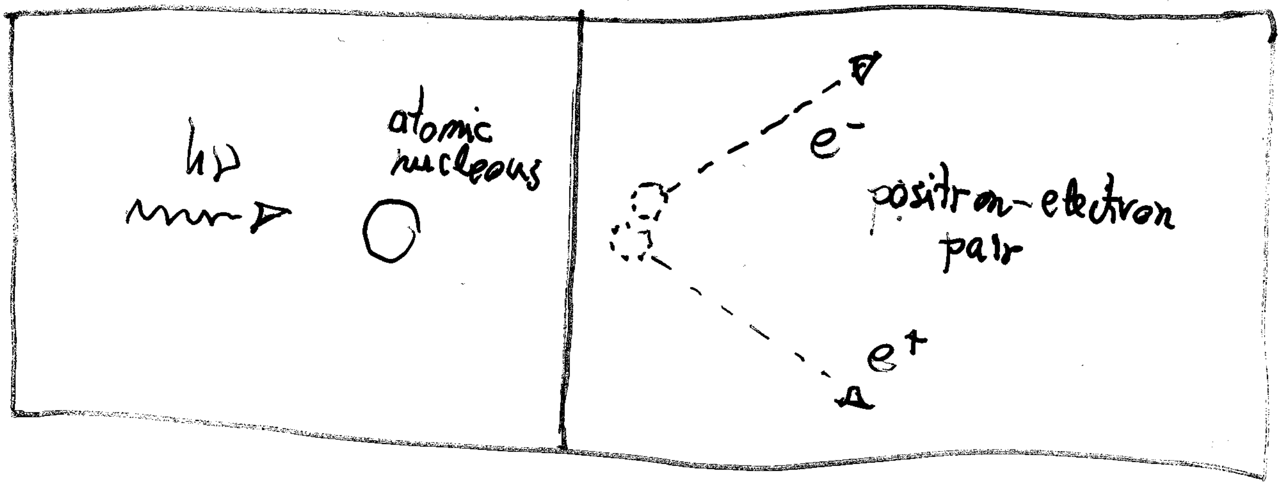


Figure 5. The Rayleigh scatter differential cross-section for iodine in the vicinity of the K-edge as determined from measurements of a dilute (3.7 mg ml^{-1}) solution of I_2 in a 100 μm diameter artificial nephron and compared to RF with ASF corrections from the RTAB database (dotted line). The error bars give the statistical error (2 s.d.) due to the Rayleigh scatter and background sources.

Pair production



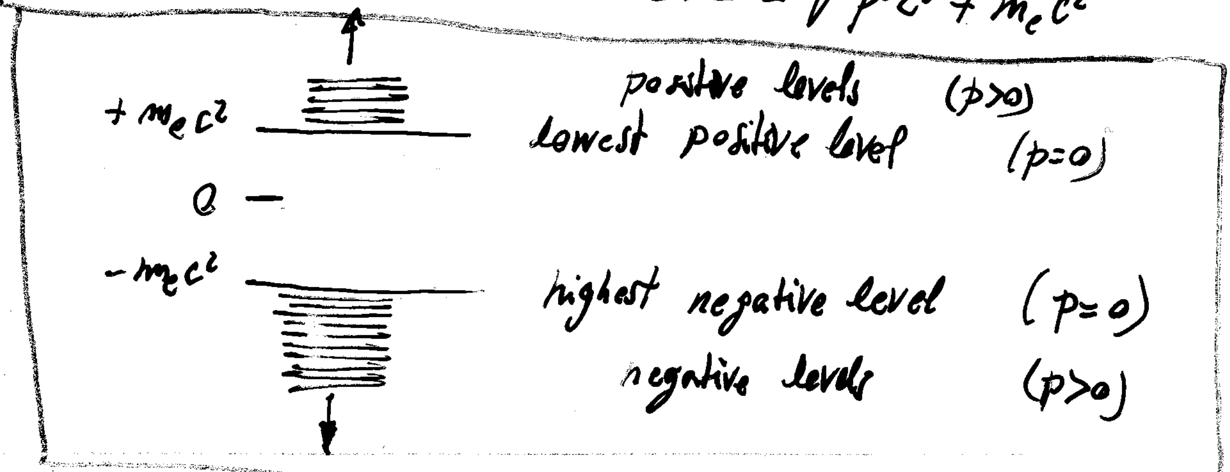
- photon energy $h\nu$ is <u>completely</u> absorbed
- the interaction requires a minimum energy $h\nu = 2 m_e c^2 \simeq 1.022$ MeV
- Energy conservation gives

$$h\nu = (T_- + m_e c^2) + (T_+ + m_e c^2)$$

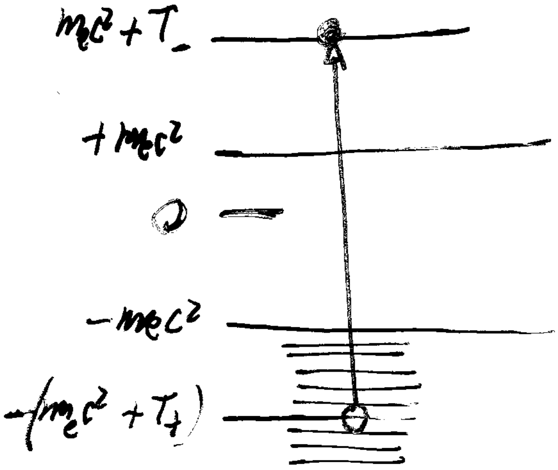
$$T_- + T_+ = h\nu - 2 m_e c^2$$

- pair production confirmed Dirac theory of the electron

$$E^2 = p^2c^2 + m_e c^2 \Rightarrow E = \pm\sqrt{p^2c^2 + m_e c^2}$$



Dirac's Empty space ⟹ formed by electrons with negative energy (filled levels)



} fully occupied negative energy states for electrons

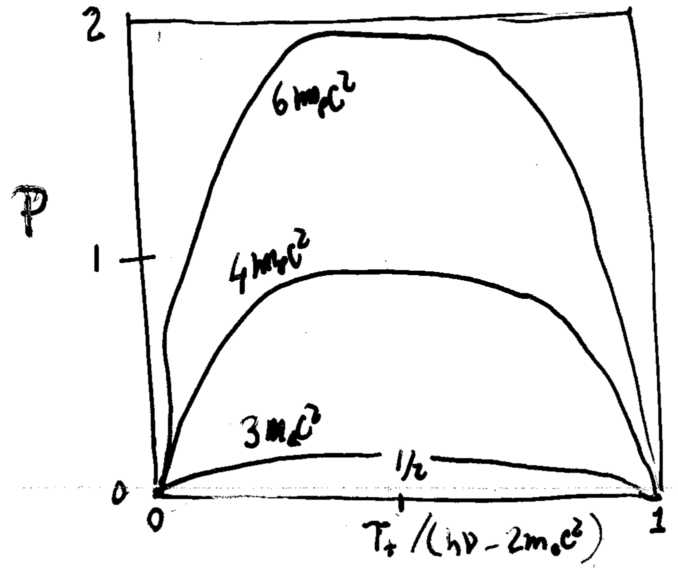
the "hole" in the negative energy levels represents the 'positron'.

Energy distribution of pairs produced electrons

$$d\,_aK = \frac{\sigma_0 Z^2 P}{h\nu - 2m_ec^2}\, dT_+$$

where

$$\sigma_0 = \frac{1}{137}\left(\frac{e^2}{m_ec^2}\right)^2 = 5{,}8\ 10^{-28}\ \frac{cm^2}{nucleus}$$



Total pair production cross section

$$ {}_a\kappa = \int d({}_a\kappa) = \sigma_0 Z^2 \int_0^{h\nu - 2m_ec^2} \frac{P\, dT_+}{h\nu - 2m_ec^2} $$

$$ \Downarrow $$

$$ {}_a\kappa = \sigma_0 Z^2 \int_0^1 P\, d\left[\frac{T_+}{h\nu - 2m_ec^2}\right] $$

$$ = \sigma_0 Z^2 \bar{P} \quad \left[\frac{cm^2}{nucleons}\right] $$

Total Attenuation Coefficient

probability of traversing a thickness x without a:

Compton collision → $e^{-\sigma_c x}$

Rayleigh collision → $e^{-\sigma_R x}$

Photoelectric collision → $e^{-\tau x}$

Pair-production collision → $e^{-\kappa x}$

A Collimated x-ray beam of initial intensity I_0 after traversing a thickness x of absorber will have a residual intensity I

$$I = I_0 e^{-\sigma_c x} e^{-\sigma_R x} e^{-\tau x} e^{-\kappa x}$$

$$= I_0 e^{-(\sigma_c + \sigma_R + \tau + \kappa) x}$$

$$= I_0 e^{-\mu x}$$

where μ is the total attenuation coefficient

$$\boxed{\mu = \sigma_c + \sigma_R + \tau + \kappa}$$

Libero cammino medio

Def: distanza media che un fotone può percorrere (in un mezzo assorbente) senza essere assorbito

[Si esprime in termini del coefficiente di assorbimento]

Spessore ottico

$$dL_\nu = \mu_\nu \, ds$$

└ spessore ottico

$$\Rightarrow L_\nu = \int_{s_0}^{s} \mu_\nu(s')\, ds' \quad \Rightarrow \begin{cases} L_\nu > 1 & \text{mezzo opaco} \\ L_\nu < 1 & \text{mezzo trasparente} \end{cases}$$

└ un punto arbitrario definisce lo zero delle scale

e^{-L_ν} → probabilità di percorrere uno spessore ottico L_ν

Calcoliamo lo spessore medio percorso

$$\langle L_\nu \rangle = \int_0^\infty L_\nu e^{-L_\nu} dL_\nu$$

$$= 1$$ (Si può dimostrare calcolando l'integrale)

$$\int_0^\infty L_\nu e^{-L_\nu} dL_\nu = (-L_\nu - 1) e^{-L_\nu} \Big|_0^\infty$$

$$= 1$$

<u>Libero cammino medio</u> ℓ_ν è data da:

$$\langle L_\nu \rangle = \ell_\nu \mu_\nu = 1$$

$$\Longrightarrow \boxed{\ell_\nu = \frac{1}{\mu_\nu}}$$

Energy absorption

primary ionization is produced by a photon when it removes an electron from an atom by
(1) photoelectric effect
(2) Compton collision

Energy of the secondary electron

Photoelectric effect $E = h\nu - E_{characteristic}$

Compton scattering $E = h\nu - h\nu'$

$\Rightarrow$ Most of the energy of the primary photon is transferred to the secondary electron

Energy is released (by secondary electrons) through two mechanisms:

(a) bremsstrahlung [small fraction ~ few %]

(b) ionization ← [Secondary ionization]
(~ 32 eV per ion-pair produced)

Example: 1 MeV electron → ~ 30.000 ionpairs

primary ionization 1 (negligible)

Secondary ionization 30 000