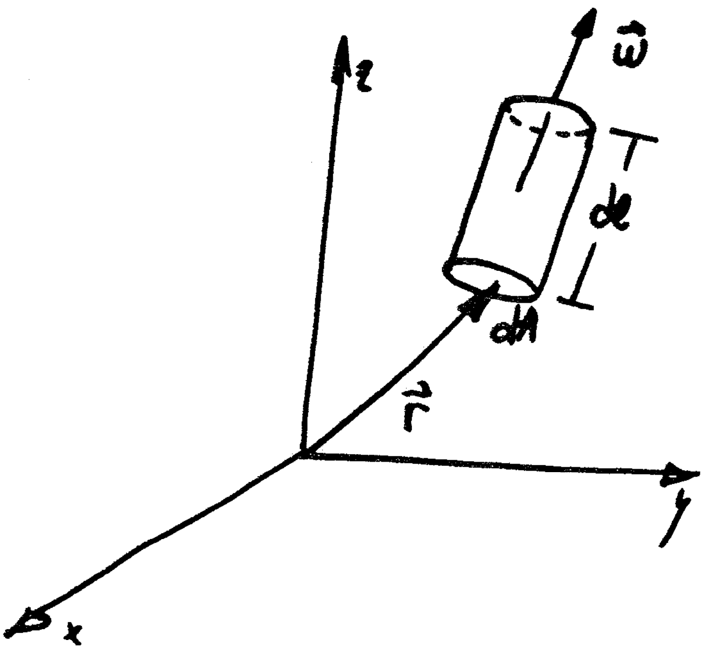
# THE BOLTZMANN TRANSPORT EQUATION FOR PHOTONS

- describes the balance between the numbers of photons of given energy and direction entering and leaving an infinitesimal cylinder




- flux $f(\vec{r},\vec{\omega},\lambda)\,d\vec{\omega}\,d\lambda$

  the number of photons with wavelengths between $\lambda$ and $\lambda + d\lambda$ and directions between $\vec{\omega}$ and $\vec{\omega} + d\vec{\omega}$ crossing a unit area per unit time.

- we use $\lambda$ in place of $E$ (immaterial)
- The net flux with specified direction and $\lambda$ leaving the cylinder per unit time is

$$f(\vec{r} + \vec{\omega}\,dl, \vec{\omega}, \lambda)\,dA - f(\vec{r}, \vec{\omega}, \lambda)\,dA$$

or

$$\vec{\omega} \cdot \nabla f(\vec{r}, \vec{\omega}, \lambda)\, dA\, dl$$

in differential form.

- Three factors contribute to this net outflow

(i) "narrow beam" attenuation in the whole volume of the cylinder

$$-\mu(\lambda)\, dl\, dA\, f(\vec{r}, \vec{\omega}, \lambda)$$

(ii) scattering of photons

$$\vec{\omega}' \rightarrow \vec{\omega}$$
$$\lambda' \rightarrow \lambda$$

$$\int_0^\infty \int_{4\pi} f(\vec{r}, \vec{\omega}', \lambda')\, k(\omega, \lambda, \vec{\omega}', \lambda')\, d\vec{\omega}'\, d\lambda'$$

probability of photon scattering ($\vec{\omega}' \rightarrow \vec{\omega}, \lambda' \rightarrow \lambda$) per unit path through the medium and per unit $d\vec{\omega}$ and $d\lambda$

Formally equivalent to

$$\frac{d\sigma}{d\vec{\omega}\, d\lambda}$$

(iii) The source

$$S(\vec{r}, \vec{\omega}, \lambda)$$

The density of photons per unit volume, per unit time per steradian and per unit $\lambda$.

# NEUTRONS (STATIONARY PROBLEM)

$$v \vec{\omega} . \nabla f(\vec{r}, v\vec{\omega}) + \frac{v f}{l_{tot}(v)} =$$

$$= \int \frac{v' \, dv'}{l_{tot}(v')} \iint f(\vec{r}, v'\vec{\Omega}') \, k(v'\vec{\Omega}' \rightarrow v\vec{\omega}) \, d\vec{\Omega}' + S$$

$$f(\vec{r}, v\vec{\omega}) \, d\vec{r} \, dv \, d\vec{\omega}$$

number of neutrons in $d^3r$ around $\vec{r}$ belonging to $dv \, d\vec{\omega}$ (travelling with speed between $v$ and $v+dv$ in a direction lying within $d\vec{\omega}$ around $\vec{\omega}$.

$$v \longrightarrow E \rightarrow \lambda$$

$$\frac{1}{l_{tot}} = \Sigma \longrightarrow \mu$$

# THREE-D PHOTON TRANSPORT EQUATION (TIME INDEPENDENT)

$$\vec{\omega}.\nabla f(\vec{r},\vec{\omega},\lambda) = -\mu(\lambda) f(\vec{r},\vec{\omega},\lambda) +$$

$$+\int_0^\infty d\lambda' \int_{4\pi} d\vec{\omega}'\, k(\vec{\omega},\lambda,\vec{\omega}';\lambda')\, f(\vec{r},\vec{\omega}',\lambda') + S(\vec{r},\vec{\omega},\lambda) \quad (1)$$

ASSUMPTIONS

- the x-ray source is constant in time, and
- is monochromatic and collimated

$$S(\vec{r},\vec{\omega},\lambda) = I_0\, \delta(\vec{\omega}-\vec{\omega}_0)\, \delta(\lambda-\lambda_0)\, \delta(\vec{u}.\vec{r} - \vec{u}.\vec{r}_0)$$

the plane $\vec{u}.\vec{r} = \vec{u}.\vec{r}_0$ defined by the point $\vec{r}_0$ and normal to $\vec{u}$.

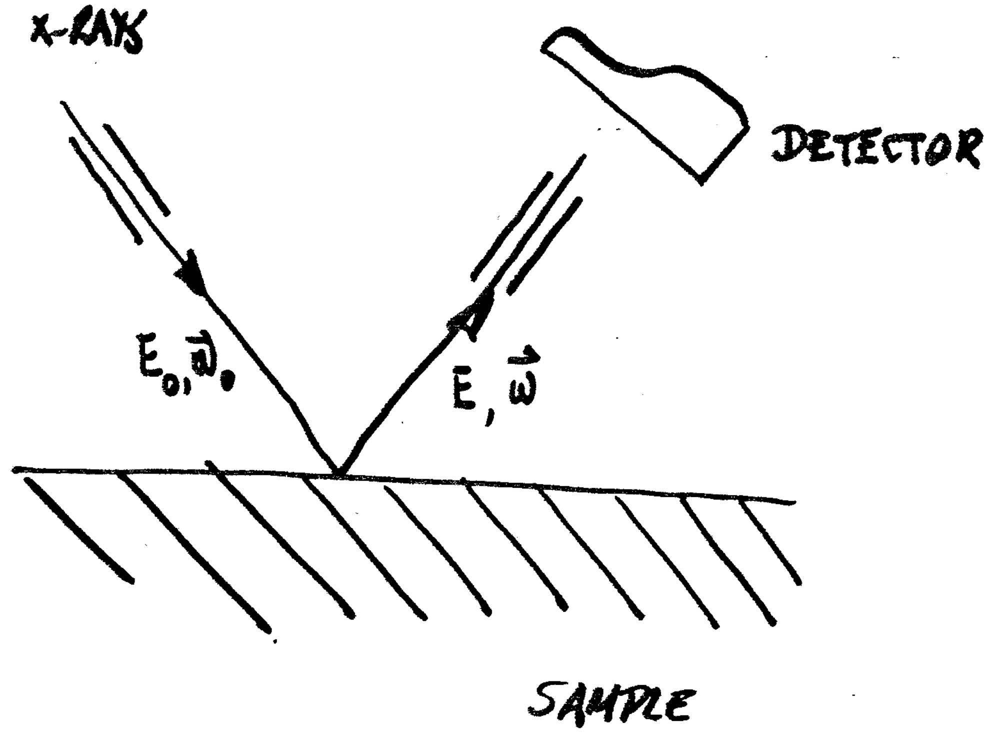
- for an infinite and homogeneous target we get that the source depends on the single space coordinate

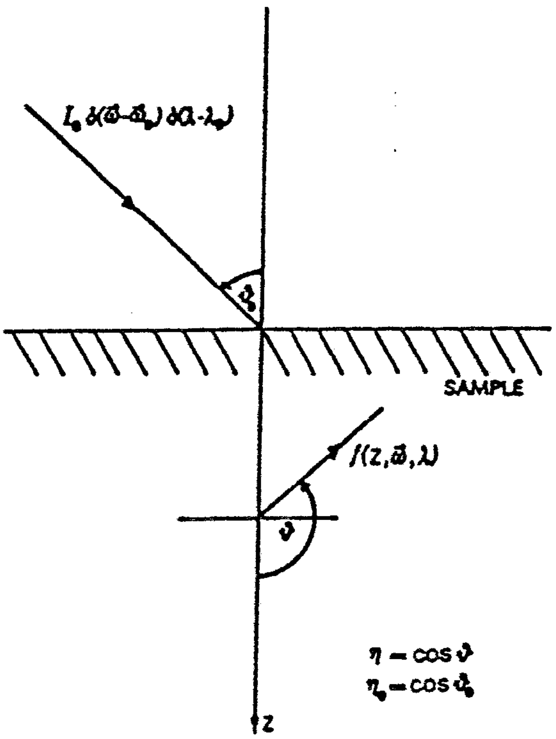
$$\vec{r}.\vec{\Omega} \quad (\text{for arbitrary } \vec{\Omega})$$

and the eqn (1) reduces to $(\eta = \omega_z)$

$$\eta \frac{\partial f(z,\vec{\omega},\lambda)}{\partial z} = -\mu(\lambda) f(z,\vec{\omega},\lambda) +$$

$$+\int_0^\infty d\lambda' \int_{4\pi} d\vec{\omega}'\, k(\vec{\omega},\lambda,\vec{\omega}';\lambda')\, f(z,\vec{\omega}';\lambda') + I_0\, \delta(z)\, \delta(\vec{\omega}-\vec{\omega}_0)\, \delta(\lambda-\lambda_0)$$




Irradiation arrangement of a homogeneous specimen of infinite thickness under the excitation of a plane monochromatic X-ray source, represented with the photon transport equation (1).